DATA JE MATRICA $A=\begin{pmatrix} 5 & 4 & 1 & 1 \\ 4 & 5 & 1 & 1 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 4 \end{pmatrix}$ * NE MORA BITI SIMETRIČNA!

QR METODOM ODREDITI SVE SOPSTVENE VREDNOSTI MATRICE A SA TAČNOŠĆU 10^{-2}

$\boxed{1a}$ Householderovom metodom najpre svodimo A u gornje-Hesenbergovu, a zatim za dobijanje Q i R koristimo Givensovu metodu.

$$x_1=\begin{pmatrix}4\\1\\1\end{pmatrix} \quad \sigma=\|x_1\|_2=4.2426$$

$$k=-\sigma\cdot \mathrm{sign}(x_1(1))=-4.2426$$

$$\beta=(\sigma\cdot(\sigma+|x_1(1)|))^{-1}=0.0286$$

$$u=x_1-k\cdot e_1=\begin{pmatrix}8.2426\\1\\1\end{pmatrix}$$

$$H=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-\beta\cdot u\cdot u^*=\begin{pmatrix}-0.9428 & -0.2357 & -0.2357\\ -0.2357 & 0.9714 & -0.0286\\ -0.2357 & -0.0286 & 0.9714\end{pmatrix}$$

$$T_1=\begin{bmatrix}1 & 0\ 0\ 0\\ 0 & \\ 0 & H\\ 0 & \end{bmatrix}$$

$$A_1=T_1AT_1=\begin{pmatrix}5 & -4.2426 & 0 & 0\\ -4.2426 & 6 & -1 & -1\\ 0 & -1 & 3.5 & 1.5\\ 0 & -1 & 1.5 & 3.5\end{pmatrix}$$

$$x_1=\begin{pmatrix}-1\\-1\end{pmatrix} \quad \sigma=1.4142,\ k=1.4142,\ \beta=0.2929,\ u=\begin{pmatrix}-2.4142\\-1\end{pmatrix}$$

$$H=\begin{pmatrix}-0.7071 & -0.7071\\ -0.7071 & 0.7071\end{pmatrix},\quad T_2=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0& & \\0&0& H & \end{pmatrix}$$

$$A_2=T_2\cdot A_1\cdot T_2=\begin{pmatrix}5 & -4.2426 & 0 & 0\\ -4.2426 & 6 & 1.4142 & 0\\ 0 & 1.4142 & 5 & 0\\ 0 & 0 & 0 & 2\end{pmatrix}=A^0$$

(*) Sada Givensovom metodom rastavljamo A^0 na Q i R. Pošto anuliramo elemente sa pod-dijagonale, koristimo izmenjene formule.

$$\alpha=\frac{|a_{kk}|}{\sqrt{|a_{kk}|^2+|a_{lk}|^2}},\quad \beta=\frac{-a_{lk}}{\sqrt{|a_{kk}|^2+|a_{lk}|^2}}$$

$a_{21}\to 0$, $k=1$, $l=2$, $\alpha=0.9881$, $\beta=0.1539$, $U_{12}=\begin{pmatrix}0.9881 & -0.1539 & 0 & 0\\ 0.1539 & 0.9881 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$

$$A_1^0=U_{12}\cdot A^0=\begin{pmatrix}9.7205 & -2.0614 & -0.3103 & 0\\ 0 & 3.5112 & 1.9920 & 0\\ 0 & 2.0160 & 2.6087 & 0\\ 0 & 0 & 0 & 2\end{pmatrix}$$

$a_{32}\to 0$, $k=2$, $l=3$, $\alpha=0.8672$, $\beta=-0.4979$, $U_{23}=\begin{pmatrix}1&0&0&0\\0&\alpha&-\beta&0\\0&\beta&\alpha&0\\0&0&0&1\end{pmatrix}$

$$A_2^0=U_{23}\cdot A_1^0=\begin{pmatrix}9.7205 & -2.0614 & -0.3103 & 0\\ 0 & 4.0488 & 3.0264 & 0\\ 0 & 0 & 1.2704 & 0\\ 0 & 0 & 0 & 2\end{pmatrix}=R_0$$

(NA POZICIJI (4,3) JE VEĆ NULA, PA NEMA POTREBE ZA JOŠ JEDNOM ITERACIJOM)

$$Q_0 = U_{12}^T \cdot U_{23}^T = \begin{pmatrix} 0.9881 & 0.1335 & -0.0766 & 0 \\ -0.1539 & 0.8569 & -0.4920 & 0 \\ 0 & 0.4979 & 0.8672 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

NAKON ŠTO SMO RASTAVILI $A^0 = Q_0 \cdot R_0$, URADIMO JEDAN (PRVI) KORAK QR METODE

$$A^1 = R_0 \cdot Q_0 = \begin{pmatrix} 9.6047 & -1.4963 & 0 & 0 \\ -1.4963 & 3.7867 & 2.0160 & 0 \\ 0 & 2.0160 & 2.6087 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

KRITERIJUM ZAUSTAVLJANJA $\| diag(A^0) - diag(A^1) \| < \varepsilon = 10^{-2}$ (U PRAKSI $\| \cdot \|_\infty$)

$$\| (5,6,5,2)^T - (9.6047, 3.7867, 2.6087, 2)^T \|_\infty = 4.6047 > 10^{-2}$$

RAČUNAMO DALJE: MATRICU A^1 RASTAVLJAMO NA Q_1 i R_1 GIVENSOVOM METODOM (*) ANULIRAJUĆI ELEMENTE PODDIJAGONALE, PA $A^2 = R_1 \cdot Q_1$ ... ITD

NAKON 5 ITERACIJA KRITERIJUM ZAUSTAVLJANJA ĆE BITI ISTINIT.

SOPSTVENE VREDNOSTI SE NALAZE NA DIJAGONALI MATRICE A^5:

[9.9988, 5.0012, 1, 2].

$\boxed{1b}$ GIVENSOVOM METODOM NAJPRE SVODIMO A U GORNJE HESENBERGOVU, A ~~ZATIM~~ ~~ZA~~ DOBIJANJE Q i R OPET KORISTIMO GIVENSOVU METODU.

ZA DOBIJANJE GORNJE HESENBERGOVE FORME: $\alpha = \frac{|a_{k,k-1}|}{\sqrt{|a_{k,k-1}|^2 + |a_{l,k-1}|^2}}$, $\beta = \frac{a_{l,k-1}}{\sqrt{|a_{k,k-1}|^2 + |a_{l,k-1}|^2}}$

$a_{31} \to 0$, $k=2$, $l=3$, $a_{k,k-1} \overset{?}{\neq} 0$ ✓

$$\alpha = \frac{a_{21}}{\sqrt{a_{21}^2 + a_{31}^2}} = 0.9701, \quad \beta = \frac{a_{31}}{\sqrt{a_{21}^2 + a_{31}^2}} = 0.2425$$

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9701 & -0.2425 & 0 \\ 0 & 0.2425 & 0.9701 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = U_{23}^T \cdot A \cdot U_{23} = \begin{pmatrix} 5 & 4.1231 & 0 & 1 \\ 4.1231 & 5.4118 & 0.6471 & 1.4552 \\ 0 & 0.6471 & 3.5882 & 1.6977 \\ 1 & 1.4552 & 1.6977 & 4 \end{pmatrix}$$

$a_{41} \to 0$, $k=2$, $l=4$, $a_{21} \overset{?}{\neq} 0$ ✓

$$\alpha = 0.9718, \quad \beta = 0.2357, \quad U_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -\beta \\ 0 & 0 & 1 & 0 \\ 0 & \beta & 0 & \alpha \end{pmatrix}$$

$$A_2 = U_{24}^T \cdot A_1 \cdot U_{24} = \begin{pmatrix} 5 & 4.2426 & 0 & 0 \\ 4.2426 & 6 & 1.0290 & 0.9701 \\ 0 & 1.0290 & 3.5882 & 1.4974 \\ 0 & 0.9701 & 1.4974 & 3.4118 \end{pmatrix}$$

$a_{42} \to 0$, $k=3$, $l=4$, $a_{21} \neq 0$ ✓

$$\alpha = 0.7276, \quad \beta = 0.6860, \quad U_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & -\beta \\ 0 & 0 & \beta & \alpha \end{pmatrix}$$

$$A_3 = U_{34}^T \cdot A_2 \cdot U_{34} = \begin{pmatrix} 5 & 4.2426 & 0 & 0 \\ 4.2426 & 6 & 1.4142 & 0 \\ 0 & 1.4142 & 5 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = A^0$$

SADA GIVENSOVOM METODOM (*) RASTAVLJAMO A^0 NA Q i R.

(MATRICA A^0 NAM SE RAZLIKUJE OD SLUČAJA 1a ZBOG MINUSA NA POZICIJAMA (2,1), (1,2), ZATO RAČUN NIJE ISTI)

$a_{21} \to 0$, $l=2, k=1$, $\alpha = 0.7625$, $\beta = -0.6470$, $U_{12} = \begin{pmatrix} \alpha & -\beta & 0 & 0 \\ \beta & \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$A_1^0 = U_{12} \cdot A^0 = \begin{pmatrix} 6.5574 & 7.1170 & 0.9150 & 0 \\ 0 & 1.8300 & 1.0783 & 0 \\ 0 & 1.4142 & 5 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$a_{32} \to 0$, $l=3, k=2$, $\alpha = 0.7913$, $\beta = -0.6115$, $U_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & -\beta & 0 \\ 0 & \beta & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$A_2^0 = U_{23} \cdot A_1^0 = \begin{pmatrix} 6.5574 & 7.1170 & 0.9150 & 0 \\ 0 & 2.3128 & 3.9107 & 0 \\ 0 & 0 & 3.2969 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = R_0$$

$$Q_0 = U_{12}^T \cdot U_{23}^T = \begin{pmatrix} 0.7625 & -0.5119 & 0.3956 & 0 \\ 0.6470 & 0.6033 & -0.4663 & 0 \\ 0 & 0.6115 & 0.7913 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^1 = R_0 \cdot Q_0 = \begin{pmatrix} 9.6047 & 1.4963 & 0 & 0 \\ 1.4963 & 3.7867 & 2.0160 & 0 \\ 0 & 2.0160 & 2.6087 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$\vdots$

NAKON 5 ITERACIJA KRITERIJUM ZAUSTAVLJANJA ĆE BITI ISPUNJEN I SOPSTVENE VREDNOSTI SU NA DIJAGONALI A^5: $[9.9888, 5.0012, 1, 2]$.

2a) Householderovom METODOM RASTAVLJAMO A NA Q i R (BEZ svođenja U GORNJE Hess.) $A = A^0$

$$X_1 = \begin{pmatrix} 5 \\ 4 \\ 1 \\ 1 \end{pmatrix} \quad \sigma = 6.5574, \; k = -6.5574, \; \beta = 0.0132, \; \mu = \begin{pmatrix} 11.5574 \\ 4 \\ 1 \\ 0 \end{pmatrix}$$

$$H = T_1 = \begin{pmatrix} -0.7625 & -0.6100 & -0.1525 & -0.1525 \\ -0.6100 & 0.7889 & -0.0528 & -0.0528 \\ -0.1525 & -0.0528 & 0.9868 & -0.0132 \\ -0.1525 & -0.0528 & -0.0132 & 0.9868 \end{pmatrix}$$

$$A_1 = T_1 \cdot A = \begin{pmatrix} -6.5574 & -6.4049 & -2.2875 & -2.2875 \\ 0 & 1.3989 & -0.1378 & -0.1378 \\ 0 & 0.0997 & 3.7156 & 1.7156 \\ 0 & 0.0997 & 1.7156 & 3.7156 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 1.3989 \\ 0.0997 \\ 0.0997 \end{pmatrix} \quad \sigma = 1.4060, \; k = 1.4060, \; \beta = 0.2536, \; \mu = \begin{pmatrix} 2.8048 \\ 0.0997 \\ 0.0997 \end{pmatrix}$$

$$H = \begin{pmatrix} -0.9950 & -0.0709 & -0.0709 \\ -0.0709 & 0.9975 & -0.0025 \\ -0.0709 & -0.0025 & 0.9975 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 1 & 0 \; 0 \; 0 \\ 0 & \\ 0 & H \\ 0 & \end{pmatrix}$$

$$A_2 = T_2 \cdot A_1 = \begin{pmatrix} -6.5574 & -6.4049 & -2.2875 & -2.2875 \\ 0 & -1.4060 & -0.2481 & -0.2481 \\ 0 & 0 & 3.7116 & 1.7116 \\ 0 & 0 & 1.7116 & 3.7116 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 3.7116 \\ 1.7116 \end{pmatrix}, \quad \sigma = 4.0873, \; k = -4.0873, \; \beta = 0.0314, \; \mu = \begin{pmatrix} 7.7989 \\ 1.7116 \end{pmatrix}$$

$$H = \begin{pmatrix} -0.9081 & -0.4188 \\ -0.4188 & 0.9081 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & H & \end{pmatrix}$$

$$A_3 = T_3 \cdot A_2 = \begin{pmatrix} -6.5574 & -6.4049 & -2.2875 & -2.2875 \\ 0 & -1.4060 & -0.2481 & -0.2481 \\ 0 & 0 & -4.0873 & -3.1086 \\ 0 & 0 & 0 & 2.6537 \end{pmatrix} = R_0$$

$$Q_0 = T_1 \cdot T_2 \cdot T_3 = \begin{pmatrix} -0.7625 & 0.6286 & 0.1439 & -0.0531 \\ -0.6100 & -0.7774 & 0.1439 & -0.0531 \\ -0.1525 & -0.0165 & -0.8823 & -0.4246 \\ -0.1525 & -0.0165 & -0.4030 & 0.9023 \end{pmatrix}$$

$$A^1 = R_0 \cdot Q_0 = \begin{pmatrix} 9.6047 & 0.9333 & 1.0974 & -0.4047 \\ 0.9333 & 1.1012 & 0.1190 & -0.0439 \\ 1.0974 & 0.1190 & 4.8998 & -1.0684 \\ -0.4047 & -0.0439 & -1.0684 & 2.3944 \end{pmatrix}$$

$$\| [9.6047, 1.1012, 4.8998, 2.3944] - [5, 5, 4, 4] \|_\infty = 4.6047 > 10^{-2}$$

CEO POSTUPAK PONAVLJAMO ~~ZA~~ MATRICU A^1.

NAKON 5 ITERACIJA, S.V. SU NA dijagonali A^5: [9.9988, 1, 5.0010, 2.0003]

UGRAĐENA funkcija [Q,R]=qr(A) VRŠI RASTAVLJANJE MATRICE A NA Q i R NA OVAJ NAČIN.

$\boxed{2b}$ GIVENSOVOM METODOM (*) RASTAVLJAMO $\overset{A^0}{A}$ NA Q i R (BEZ svođenja u Gornje-Hess.)

$a_{21} \to 0$, $l=2, k=1$, $\alpha = 0.7809$, $\beta = -0.6247$, $U_{12} = \begin{pmatrix} \alpha & -\beta & 0 & 0 \\ \beta & \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$A_1 = U_{12} \cdot A = \begin{pmatrix} 6.4031 & 6.2470 & 1.4056 & 1.4056 \\ 0 & 1.4056 & 0.1562 & 0.1562 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 4 \end{pmatrix}$$

$a_{31} \to 0$, $l=3, k=1$, $\alpha = 0.9880$, $\beta = -6.1543$, $U_{13} = \begin{pmatrix} \alpha & 0 & -\beta & 0 \\ 0 & 1 & 0 & 0 \\ \beta & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$A_2 = U_{13} \cdot A_1 = \begin{pmatrix} 6.4807 & 6.3264 & 2.0059 & 1.6973 \\ 0 & 1.4056 & 0.1562 & 0.1562 \\ 0 & 0.0241 & 3.7352 & 1.7532 \\ 1 & 1 & 2 & 4 \end{pmatrix}$$

$a_{41} \to 0$, $l=4, k=1$, $\alpha = 0.9883$, $\beta = -0.1525$, $U_{14} = \begin{pmatrix} \alpha & 0 & 0 & -\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta & 0 & 0 & \alpha \end{pmatrix}$

$$A_3 = U_{14} \cdot A_2 = \begin{pmatrix} 6.5574 & 6.4049 & 2.2875 & 2.2875 \\ 0 & 1.4056 & 0.1562 & 0.1562 \\ 0 & 0.0241 & 3.7352 & 1.7582 \\ 0 & 0.0235 & 1.6707 & 3.6944 \end{pmatrix}$$

$a_{32} \to 0$, $l=3, k=2$, $\alpha = 0.9999$, $\beta = -0.0171$, $U_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & -\beta & 0 \\ 0 & \beta & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$A_4 = U_{23} \cdot A_3 = \begin{pmatrix} 6.5574 & 6.4049 & 2.2875 & 2.2875 \\ 0 & 1.4058 & 0.2202 & 0.1863 \\ 0 & 0 & 3.7320 & 1.7562 \\ 0 & 0.0235 & 1.6707 & 3.6944 \end{pmatrix}$$

$a_{42} \to 0$, $l=4, k=2$, $\alpha = 0.9999$, $\beta = -0.0167$, $U_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -\beta \\ 0 & 0 & 1 & 0 \\ 0 & \beta & 0 & \alpha \end{pmatrix}$

$$A_5 = U_{24} \cdot A_4 = \begin{pmatrix} 6.5574 & 6.4049 & 2.2875 & 2.2875 \\ 0 & 1.4060 & 0.2481 & 0.2481 \\ 0 & 0 & 3.7320 & 1.7562 \\ 0 & 0 & 1.6668 & 3.6907 \end{pmatrix}$$

$a_{43} \to 0$, $l=4, k=3$, $\alpha = 0.9131$, $\beta = -0.4078$, $U_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & -\beta \\ 0 & 0 & \beta & \alpha \end{pmatrix}$

$$A_6 = U_{34} \cdot A_5 = \begin{pmatrix} 6.5574 & 6.4049 & 2.2875 & 2.2875 \\ 0 & 1.4060 & 0.2481 & 0.2481 \\ 0 & 0 & 4.0873 & 3.1086 \\ 0 & 0 & 0 & 2.6537 \end{pmatrix} = R_0$$

$$Q_0 = U_{12}^T \cdot U_{13}^T \cdot U_{14}^T \cdot U_{23}^T \cdot U_{24}^T \cdot U_{34}^T = \begin{pmatrix} 0.7625 & -0.6286 & -0.1439 & -0.0531 \\ 0.6100 & 0.7774 & -0.1439 & -0.0531 \\ 0.1525 & 0.0165 & 0.8923 & -0.4246 \\ 0.1525 & 0.0165 & 0.4030 & 0.9023 \end{pmatrix}$$

$$A^1 = R_0 \cdot Q_0 = \begin{pmatrix} 9.6047 & 0.9333 & 1.0974 & 0.4047 \\ 0.9333 & 1.1012 & 0.1190 & 0.0439 \\ 1.0974 & 0.1190 & 4.8998 & 1.0694 \\ 0.4047 & 0.0439 & 1.0694 & 2.3944 \end{pmatrix}$$

POSTUPAK PONAVLJAMO ~~ZA~~ A^1. NAKON 5 ITERACIJA krit. zaustavljanja će biti ispunjen. S.vrednosti su na dijagonali $A5$

[9.9988, 1, 5.0010, 2.0003]